

# Design of a Prestressed Concrete Stress Ribbon Bridge

Aditya V. Jog<sup>1</sup>, D. M. Joshi<sup>2</sup>

<sup>1</sup>P.G. Student, Saraswati College of Engineering, Kharghar, Maharashtra, India.  
 adityavjog@gmail.com

<sup>2</sup>Assistant Professor, Saraswati College of Engineering, Kharghar, Maharashtra, India.  
 dmjoshi123@yahoo.com

**Abstract:** A Stress Ribbon Bridge is a tension structure, which has similar geometry to that of a simple suspension bridge. This type of bridge is mainly used for pedestrian or cycling traffic. It is a slender structure, in which the suspended cables are embedded in the slab deck, following a catenary arc between the supports. The system is then prestressed using another set of tensioning cables to provide flexural rigidity to the structure. This paper deals with the static behavior of stress ribbon bridges. The structural governing equations are studied and presented. The equations are then integrated for a particular case of a single span bridge, accounting for various loading conditions and prestressing forces. An example of the application is shown based on the presented formulation.

**Key words:** preliminary design, post-tensioning, static behavior, stress ribbon bridge.

## INTRODUCTION

The cable has always been recognized as a very efficient load-bearing material. In one of its most simple forms, the bearing ropes are used to support cross members allowing the passage of pedestrians across the catenary itself. This primitive idea of using the bearing cables for passage of pedestrians was recovered in the 1960's through the concept of Stress Ribbon Bridges.

This concept was first introduced by German engineer **Ulrich Finsterwalder**. This type of bridge uses the theory of a catenary transmitting loads via tension in the deck to abutments which are anchored to the ground. The first Stress Ribbon bridge was constructed in 1960's in Switzerland.

The construction practice involves embedding the bearing cables in a relatively thin band of concrete and then, through additional prestressing of the cable, setting the so created inverted concrete arch under uplift pressures, which creates a compressive state of stress in the arch. Thus, additional live loads applied later can be taken up without excessive deformation of the cable, thanks to the effective bending rigidity of the arch acquired through the compressive state of stress in the arch.

This type yields slender, aesthetical and almost maintenance free structures. Such structures are erected independently from existing terrain and thus have a minimum impact on the environment during construction. On the other hand, one of the major drawbacks of this structural system is the existence of large horizontal tension forces that must be resisted by the abutments.

Since its introduction in the 1960's, a number of bridges

have been constructed across the world. Few of the outstanding examples include bridges in Republic of Czechoslovakia [1], Sacramento River, United States [2], Vltava River [3]. Though the existing literature on this subject is not very extensive, the most important contributions concerning the analysis and design are those of Prof. Jiri Strasky [4].

This paper deals with the statical behavior of the stress ribbon bridges. The structural governing equations are studied and presented first. The equations are then integrated for a particular case of single span bridge accounting for distributed loads, concentrated loads, prestressing, and temperature effects. The formulation is then applied to the preliminary design of an 80m span bridge and comparison with numerical results [5] is made.

## GOVERNING EQUATIONS

### Initial geometry

After the placing of precast segments, the geometry of the stress ribbon bridge is that of a cable suspended under its own weight. With reference to Fig 2, let "q<sub>0</sub>" be the self-weight per unit length of the deck and "H<sub>0</sub>" the horizontal force acting on the cables. It can be assumed that when subjected to uniform continuous load "q", a parabolic shape of the deck is formed instead of catenary. "H<sub>0</sub>" in parabola and catenary are practically equal for the given sag/span (d<sub>0</sub>/l) ratio (Fig 1, Petersen, 1993).

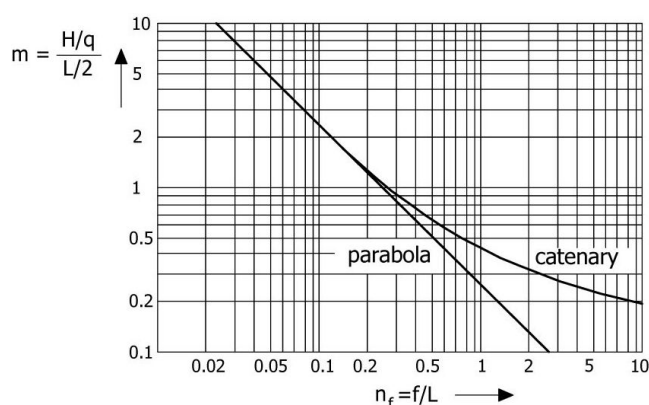


Fig 1. Diagram of dimensionless parameters for parabola and catenary (Petersen, 1993.)

For the case of supports at equal heights, the vertical equilibrium equation [6] and its integral gives,

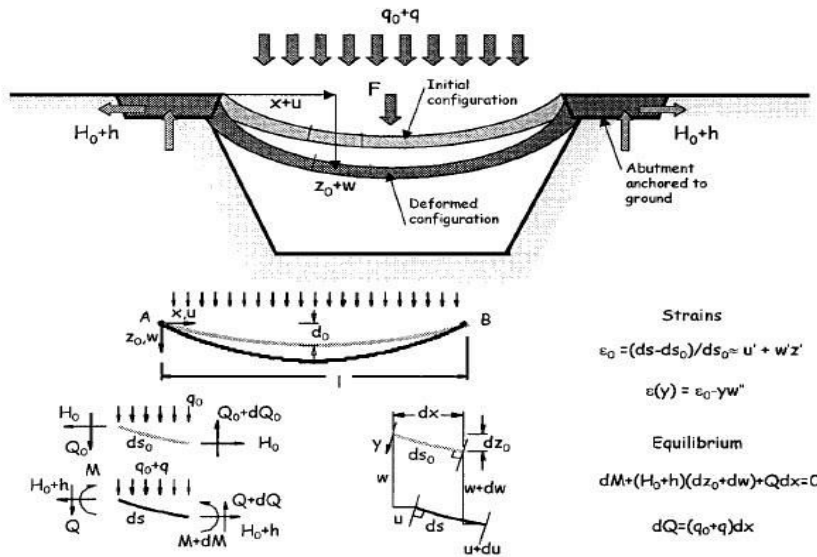


Fig 2. Initial Geometry and Deformed Configuration

$$H_0 Z_0^n = -q_0 ; Z_0(x) = \frac{q_0}{2H_0} x(l-x) ; d_0 = \frac{q_0 l^2}{8H_0} \quad (1)$$

where  $Z_0$  = initial geometry profile of the ribbon;  $l$  = length of the span; and  $d_0$  = sag of the cable.

### Equations for single span Stress Ribbon Bridge

Once the joints are connected, the flexural stiffness of the deck must be included in the equilibrium equation. Under action of additional vertical load  $q(x)$  and temperature increase  $\Delta t$ , the bridge deforms from the reference geometry (Fig 2). According to the common shallow arch hypothesis [7],  $(Z_0')^2 \ll 1, (w')^2 \ll 1, u' \ll 1$ , the equilibrium equations of differential element of bridge are :

$$Q + M' + (h + H_0)(Z_0' + w') = 0 \quad (2)$$

$$Q' = q + q_0 \quad (3)$$

Small deformations, material linearity, and the usual Navier-Bernoulli hypothesis that plane sections remain plane and perpendicular to the centre line,  $h$  and  $M$  can be expressed as follows:

$$h = EA(u' + Z_0'w') \quad (4)$$

$$M(x) = -EIw'' \quad (5)$$

The increase in horizontal force of the ribbon is given by the compatibility equation for a suspended cable [6] :

$$h = \frac{EA}{l} \left( u_B - u_A + w_B'Z_0'B - w_A'Z_0'A + \frac{q_0}{H_0} \int_A^B w dx \right) - EA\alpha\Delta t \quad (6)$$

where  $EA$  represents axial stiffness of the ribbon for the current structure situation and  $\alpha\Delta t$  represents non-mechanical deformation due to the uniform increase in the temperature.

Equations (2), (5) and (6) are those that govern the structural behavior of stress ribbon bridge and should be solved by introducing the corresponding boundary conditions [6].

With reference to Fig 2, the governing equations for the case of a symmetrically distributed load  $q$ , concentrated symmetrical load  $F$ , and uniform rise in temperature  $\Delta t$ , the parameters  $\alpha_0, \alpha_1$  are defined as:

$$\alpha_0 = \frac{\frac{F}{l} + q - q_0 \frac{h}{H_0}}{2(H_0 + h)} , \alpha_1 = \frac{F}{2(H_0 + h)} \quad (7)$$

Thus, it can be shown that  $w$  can be given by [5]:

$$w(x) \approx \frac{\alpha_0}{\gamma} (exp^{-\gamma x} - 1) + \frac{\alpha_1(1 - \cosh(\gamma x))}{\gamma \sinh(\frac{\gamma l}{2})} + \frac{\alpha_0 - \alpha_1}{l} x(l-x) + \alpha_1 x$$

$$0 \leq x \leq \frac{l}{2} \quad (8)$$

where the approximation  $\frac{\gamma l}{2} = 1$  has been made.

Now, introducing equation (8) in (6) and integrating,

$$h = \frac{EA}{l} \frac{q_0}{H_0} \left( \frac{2\alpha_0 + \alpha_1}{12} l^2 + 2 \frac{\alpha_0 - \alpha_1}{\gamma^2} - \frac{\alpha_0 l}{\gamma} \right) - EA\alpha\Delta t \quad (9)$$

Finally differentiating (8) twice and substituting into (5),  $M$  is given as:

$$M(x) = EI \left( 2 \frac{\alpha_0 - \alpha_1}{l} - \alpha_0 \gamma exp^{-\gamma x} + \alpha_0 \gamma \frac{\cosh(\gamma x)}{\sinh(\frac{\gamma l}{2})} \right)$$

$$0 \leq x \leq \frac{l}{2} \quad (10)$$

The equations (8) and (10) are then solved for boundary conditions  $x = 0$  and  $x = l/2$ , the following expressions are obtained:

$$M(0) = EI \left( 2 \frac{\alpha_0 - \alpha_1}{l} - \alpha_0 \gamma \right)$$

$$M(l/2) = EI \left( 2 \frac{\alpha_0 - \alpha_1}{l} - \alpha_1 \gamma \right)$$

$$w \left( \frac{l}{2} \right) = (\alpha_0 + \alpha_1) \left( \frac{l}{4} - \frac{1}{\gamma} \right) \tag{11}$$

**Introduction of Prestressing**

A new set of cables are used for prestressing and tension is introduced by anchoring them to the abutments. The effect of prestressing is only the deviation forces. Assuming a constant prestressing force  $P_0$  (prestressing losses due to friction and anchorage slip are neglected), the equilibrium equation is :

$$Q + M' + (h + H_0)(Z_0' + w') = 0 \tag{12}$$

The solution to equation (12) can be obtained by substituting  $h = h - P_0$  in the expression (7) and (8) and then using formulae (9) – (11).

**PRELIMINARY DESIGN OF STRESS RIBBON BRIDGE**

As an example of the application of the presented formulation, preliminary design of 80 meter long, 5 meter wide stress ribbon pedestrian bridge is shown.

**Postulates**

The use of stress ribbon bridge for pedestrian traffic poses the restriction of maximum slope to 8%, which shows a shallowness ratio equal to 1/50. Thus for an 80 m span stress ribbon bridge, the maximum sag must be around 1.6 m. In this problem, the sag  $d_0$  is roughly taken as 2.0 m as post-tensioning will yield a value closer to the required one.

**Data for Analysis**

- $l = 80m$
- $A_c = 1.00m^2$
- $E_c = 35000 MPa$
- $I = 0.0076 m^4$
- $q_0 = 25.00 KN/m$
- $P = 21000 KN$  (after initial losses)

- $d_0 = 2.0m$
- $A_{s1} = 0.01344m^2$
- $E_s = 2000000 MPa$
- $\Delta t = -10^\circ C$
- $q_g = 6.0 KN/m$

- $H_0 = 10300 KN$
- $A_{s2} = 0.01596 m^2$
- $f_{ck} = 35 MPa$
- $\alpha = 10^{-5} \text{ } ^\circ C^{-1}$
- $q = 20.0 KN/m$

The preliminary section of the bridge deck is shown in the Fig 3 which also gives some properties of the materials used.

**Mechanical Characteristics**

The proposed section yields  $A_c = 1.0 m^2$  and  $I_c = 0.0077 m^4$ . The dead load at initial stage is taken equal to the concrete weight. Additional permanent load due to the handrail and around 4 cm of pavement is roughly approximated around 6.0 kN/m. A distributed live load of 4.0 kN/m<sup>2</sup> is considered, and uniform temperature variation of 10°C is also considered.

After the precast segments are put into place and concreted, the weight of the ribbon is resisted by the bearing cables alone.

According to (1), the initial tension in the cables is estimated as  $H_0 = 10.00 MN$ . The criterion for dimensioning of bearing cables is to limit the initial stress  $H_0/A_{s1}$  to  $0.40f_{max}$  so that problems of relaxation and fatigue are eliminated. The area of bearing cables is determined as  $A_{s1} = 13,340 mm^2$ , corresponding to four tendons of 24 strands 15.2 mm each.

The post tensioning force  $P_0 = 20.00 MN$  (after initial losses) and cross-sectional area of post tensioning cables  $A_{s2} = 15,960 mm^2$  (corresponding to six tendons of 19 strands of 15.2 mm each).

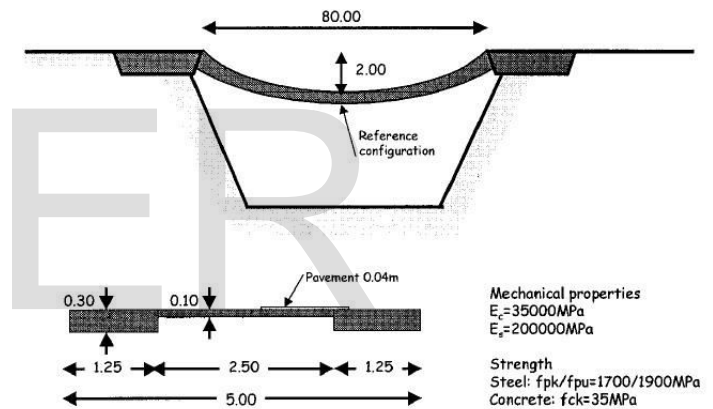


Fig 3. Preliminary dimensions of 80 m span bridge

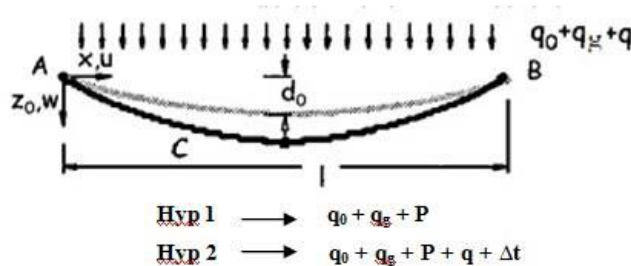


Fig 4. Loading Conditions

**RESULTS**

		$h$ KN	$M_A$ KNm	$M_C$ KNm	$w_C$ m
Hyp 1	Anal.	-15896	1006.9	-116.7	-0.278
Time $t_0$	Num.	-15928	999.2	-115.4	-0.275
Hyp 2	Anal.	-7787.5	866.4	-79.5	-0.199
Time $t_0$	Num.	-7823.7	860.0	-78.6	-0.196

**CONCLUSIONS**

The characteristics of structural behavior of prestressed concrete stress ribbon bridge have been presented. The first design of 80-m span, 5-m wide stress ribbon pedestrian bridge is shown. Equilibrium and compatibility differential equations integrated for certain load condition have been studied and presented. The expressions derived allow the evaluation of the structural response under the action of loading, post-tensioning and temperature variations. The values of horizontal forces, bending moments and vertical movements have been obtained have been developed by the formulation presented above according to loading conditions shown in Fig 4. The results are then compared with the numerical results obtained by a computer program developed by the authors based on finite element method [8].

Thus the formulas developed above prove as a helpful tool in the preliminary design of a Stress Ribbon Bridge. Based on this preliminary design, the final design of the bridge can be made addressing various loading conditions and dynamic factors such as aerodynamic stability and pedestrian-excitation.

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